## NAVORD REPORT

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HEAT TRANSFER TO DRY ICE SPHERES SUBJECTED TO SUPERSONIC AIR FLOW

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U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

## Aerodynamics Research Report 16

## HEAT TRANSFER TO DRY ICE SPHERES SUBJECTED TO SUPERSONIC AIR FLOW

Prepared by:

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ABSTRACT: Heat-transfer rates to a 5 cm diameter dry ice sphere for Mach numbers 1.86, 2.87, and 4.25 were determined from ablation experiments. Results are compared with heat-transfer rates to a non-evaporative model subjected to the same flow conditions. Heat-transfer rates to the subliming model calculated from mass ablation rates were found to be two to three times as great as heat-transfer rates to non-evaporative models when only convection was considered. This report shows that heat conduction effects internal to the dry ice model are significant, especially at the highest Mach number tested, where the greatest change in environmental pressure occurred. Other factors contributing to the large heat-transfer rates in the subliming models are: (1) change in model shape during blow; (2) increase in surface roughness during blow; and (3) loss of mass without heat transfer through fragmentation.

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Missiles flown at high speeds encounter severe aerodynamic heating which may cause serious damage to the vehicle. The use of ablating materials as a scheme to reduce these high temperatures is receiving particular attention. For this reason, investigations concerning the behavior of an ablating substance, dry ice, under the conditions of supersonic flow, have been conducted at the U. S. Naval Ordnance Laboratory under Task Number NO 502-825/51014/01. This report utilizes experimental mass transfer data, obtained by K. H. Gruenewald in the NOL wind tunnel, for the consideration of heat transferred from air to a solid carbondioxide sphere subjected to supersonic flow.

The author is indebted to Dr. K. H. Gruenewald and Mr. I. Korobkin for their assistance in the preparation of this report.

W. W. WILBOURNE Captain, USN Commander

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## CONTENTS

Introduction	on	1				
	ve Heat Transfer to Dry Ice Spheres	1				
Heat Trai	nsfer to a Non-Evaporative Hemisphere	4				
Results and	Discussion	4				
Conclusions	,	7				
References		9				
Appendix A		10				
ILLUSTRATIONS						
Figure 1.	Surface temperature of sublimation versus angular position on a dry ice sphere					
Figure 2.	Heat of sublimation for solid CO2					
Figure 3.	Heat of sublimation over a dry ice sphere in supersonic flow					
Figure 4.	Dry ice sphere, before and after blowing					

## SYMBOLS

A	= surface area involved in heat-transfer process
a	= radius of sphere
В	= constant
С	= specific heat
D	= diameter of sphere
	= convective heat transfer
. <b>H</b>	= heat of sublimation
Ħ	= average heat of sublimation
	= Bessel function
k	= thermal conductivity
K	·
	= thermal diffusivity = $k/\rho c$
m	= mass
<sup>m</sup> i	- mass at beginning of blow
m <sub>f</sub>	= mass at end of blow
p	= static pressure
$p_{\infty}$	= free-stream static pressure
$P_{\gamma \gamma}(\mu)$	- Legendre function of the first kind (zonal harmonic)
q '	= dynamic pressure
q	= time rate of heat transfer per unit area
Q	- total time rate of heat transfer
r	- distance from center of sphere
t	= time
T	= temperature
$\mathbf{T_1}$	= constant (initial temperature of dry ice sphere)
T <sub>e</sub>	= equilibrium temperature
_	

- Tw = surface temperature
- u = steady-state temperature over sphere surface
- w = variable temperature within sphere
- v = u + w = total temperature
- x = distance from stagnation point in direction of flow
- $\alpha_n = \text{root of } J_{n+1/2}^{(\alpha r)}$
- B = gradient of local velocity evaluated at stagnation point
- e angular position on sphere measured away from stagnation point
- $\rho$  = density
- W = kinematic viscosity

## HEAT TRANSFER TO DRY ICE SPHERES SUBJECTED TO SUPERSONIC AIR FLOW

### INTRODUCTION

- 1. Missiles which are to fly at supersonic and hypersonic speeds will encounter severe aerodynamic heating. Various schemes have been proposed for coping with the heating problem and the principle of surface ablation has received particular attention. Rather than attempting to prevent deterioration of the missile surface due to heating, it may be advantageous to allow ablation to occur on the surface in order to protect the remainder of the vehicle. Such an approach seems quite feasible because of the vast amounts of energy which materials can absorb when undergoing a change of phase. In effect, melting, evaporation, and sublimation represent constant temperature heat sinks. For these reasons investigations concerning the behavior of an ablating substance under the conditions of supersonic flow have been conducted at the Naval Ordnance Laboratory.
- 2. Results of experimental research at NOL pertaining to the drag and rate of sublimation of dry ice models subjected to supersonic flow have been published by Gruenewald, reference (a). The present report utilizes Gruenewald's data for the consideration of heat transferred from the air to the subliming, solid carbon-dioxide and includes the effects of conduction within the models. These results are compared with the heat transfer obtained using non-evaporative models.

## Convective Heat Transfer to Dry Ice Spheres

3. Assuming that only convective heat-transfer effects are present, the time rate of heat transfer from the air to a dry ice model can be determined from ablation considerations by the following equation:

$$Q = \frac{\Delta m}{\Delta t} \cdot \bar{H}$$
 (1)

where Q = total rate of heat transfer to the sphere;  $\triangle m/\triangle t$  = rate of ablation; and H = average latent heat of sublimation over the sphere.

4. Ablation rates per square centimeter of cross-sectional area for 5 cm diameter dry ice spheres as determined from Gruenewald's data are given below:

Mach Number	Ablation Rate - gram/sec $cm^2$
1.86	0.0448
2.87	0.0185
4.25	0.00829

The average latent heat of sublimation can be expressed as a function of mass by

$$\overline{H}(m) = \frac{1}{m_{f}-m_{i}} \int_{m_{i}}^{m_{f}} H(m) dm \qquad (2)$$

where H(m) is the latent heat associated with each ablated particle. Therefore the integral in Equation (2) represents the total heat of ablation.

- The local heats of sublimation depend upon the temperatures of the particular particles of mass involved, but these temperatures as well as other physical properties of the ablating substance are difficult to measure. is not known how the ablating process influences these properties. However, the temperature at which sublimation occurs is a function of the vapor pressure of CO2. If the concentration of CO2 vapor in the vicinity of the sphere were 100 percent, then the vapor pressure would be identical to the static pressure on the sphere. It has been assumed for the purposes of this report that the CO2 vapor pressures and the model static pressures are identical; therefore, it is necessary to know the static pressure distribution around the sphere for the Mach numbers being considered. pressure distribution data have been obtained from the work of Korobkin, reference (b). Sublimation temperatures corresponding to the static pressures on the upstream hemisphere were determined from the Mandbook of Physics and Chemistry, reference (c), and are shown in Figure 1. Undoubtedly, the true sublimation temperatures are lower than the values used in this analysis. For instance, if the vapor pressures of CO2 were 80 percent of the static pressure, the surface temperature would be lowered by approximately  $3^{\circ}$ K at the stagnation point and by  $1^{\circ}$ K at  $\Theta = 90^{\circ}$ . Therefore, the results may be regarded as a limiting condition.
- 6. The calculated sublimation temperatures can be approximated by the empirical equation:

$$T = A_0 + A_1 \cos \theta \tag{3}$$

where  $A_{\rm O}$  and  $A_{\rm I}$  are constants for a given Mach number and  $\Theta$  is the angle shown in Figure 1.

7. Figure 2 is a plot by Gruenewald (unpublished) of the latent heat of sublimation of CO2 as a function of temperature. For  $163^{\circ}\text{K} \leq T \leq 193^{\circ}\text{K}$  theoretical values of H were computed from the equation,

$$H = 6.440 + 1.73 \times 10^{-3} T - 1.78 \times 10^{-5} T^2$$
 (4)

given by Kelly (reference d). Plank and Kuprianoff, reference (e), give theoretical values of H for  $140^{\circ}\text{K} \le T \le 195^{\circ}\text{K}$ . Using a Clausius-Claperon modification for an imperfect gas, other values of H are given for  $173^{\circ}\text{K} \le T \le 216^{\circ}\text{K}$ . Experimental points obtained by Eucker and Donath, Maass and Barnes, and Kuenen and Robson are given in reference (e). Since these points fall between the computed theoretical values, they have been taken as the basis for the solid curve in Figure 2 and used in the heat-transfer analysis. Combining the information of Figures 1 and 2, the variation in the local heat of sublimation around the hemisphere was determined and is shown in Figure 3.

8. Gruenewald, reference (a), reported only gross mass losses on the ablating sphere, and did not indicate the spherical distribution of these losses. Therefore, it is impossible to determine  $H_{(m)}$  as specified by Equation (2). Fortunately, the variation of H along the sphere is small, Figure 3, and it is safe to assume that H can be determined for each Mach number on a purely geometrical basis, d.e.,

$$\bar{H} \approx \frac{1}{A} \int_0^A H dA$$
 (5)

Furthermore, Gruenewald has indicated that no visible changes occurred on the rear of the sphere nor in the model diameter normal to the flow during his wind tunnel tests. All of the ablation appeared to be restricted to the upstream half of the sphere. Therefore, the following values of H were determined from Equation (5) for the front hemisphere.

Average Heat of Sublimation for a Hemisphere

Mach Number	$\frac{\vec{H}(cal/gram)}{}$
1.86	138.9
2.87	140.6
4.25	142.2

All heat-transfer calculations using Equation (1) were performed for only the upstream hemisphere.

Heat Transfer to a Non-Evaporative Hemisphere

- 9. Because only the upstream hemisphere of the dry ice models appeared to be subject to sublimation, a direct comparison between the heat-transfer results of the present analysis and previously published work on heat transfer to non-ablating hemispheres can be made.
- 10. The time rate of heat transfer to a non-evaporative body is obtained from the equation

$$Q = h A(T_e - T_W)$$
 (6)

Where h is the heat-transfer coefficient; A the surface area involved in the heat-transfer process;  $T_e$  the equilibrium temperature; and  $T_w$  the surface temperature.

- 11. Wall temperatures were taken to be the same as those of the dry ice spheres and equilibrium temperatures were computed using values of  $T_{\rm e}/T_{\rm O}$  as given by Korobkin, reference (b).
- 12. Since local flow properties for the various Mach numbers are known from the experimental test conditions, reported in reference (a), laminar heat-transfer coefficients for non-evaporative hemispheres at each Mach number were obtained from

$$F(\theta) = \frac{\frac{hD}{k}}{\left(\frac{BD^2}{\gamma}\right)^{1/2}}$$
 (7)

given by Korobkin, reference (b). Heat-transfer rates were then calculated from Equation (6).

#### RESULTS AND DISCUSSION

13. The over-all heat-transfer rates for both 5 cm diameter dry ice hemispheres as determined from Equation (1) and for the corresponding non-evaporative hemisphere in laminar flow are given in the following table:

# Over-all Heat-Transfer Rates for Dry Ice and for Non-Evaporative Hemispheres

## Mach Number

## Over-all Q in Kcal/hr\*

	Dry Ice	Non-Evaporative
1.86	440	129
2.87	184	86
4.25	83	40

As can be seen from the above table, the amount of heat transferred to the dry ice spheres was found to be two to three times as great as the heat-transfer calculated for non-evaporative bodies. Calculations which have been performed for transpiration cooling by Smith, reference (f), and others indicate that injection and diffusion effects tend to reduce the heat transfer, not to increase it. A possible explanation of the discrepancy is the effect of conduction within the sphere.

- 14. Prior to testing in the wind tunnel, the sphere is uniformly at the sublimation temperature corresponding to atmospheric pressure. In the tunnel, sphere surface pressures are less than atmospheric and sublimation will occur at a lower surface temperature. Consequently, heat will be transferred from the high temperature interior of the sphere to the exterior causing additional sublimation. In order to make an estimate of the heat conduction effects, a solid dry ice sphere with an initial temperature,  $T_1$  and a surface temperature distribution  $F(\theta)$ , Equation (3) was examined.
- 15. The surface conditions were considered time independent. The total temperature was taken to be equal to v = u + w, where u is the solution of the steady temperature problem; and w is the solution of the heat flow through a sphere with a given initial temperature, reference (g). From Appendix A, Equation (11), the total temperature is

$$V = A_0 + \frac{A_1 r}{a} \cos \theta + \sum_{\alpha_{0i}} \left[ A_{0\alpha} e^{-K\alpha_{0i}^2 t} (\alpha_{0i} r)^{-1/2} J_{1/2} (\alpha_{0i} r) \right] + \sum_{\alpha_{1i}} \left[ A_{1\alpha} e^{-K\alpha_{1i}^2 t} (\alpha_{1i} r)^{-1/2} J_{3/2} (\alpha_{1i} r) \cos \theta \right]$$

<sup>\*</sup>Values are the average taken from a number of blows.

Taking t = 40 seconds (the average length of blow in wind tunnel for dry ice tests),  $\rho = 1.52~\rm g/cm^3$  (determined from dry ice used in tests), C = 0.31 cal/g°C (reference h) and k = .00108 cal/cm°C sec, (reference i), and a = 2.5 cms, the heat transfer at the surface was determined from the equations

 $q = -k \left( \frac{dv}{dr} \right)_{r=a}$  and  $Q = \int_0^A q dA$ 

The results are included in the table below.

Heat Transfer by Conduction in 5 cm Diameter
Dry Ice Hemisphere

Mach Number		1.86	2.87	4.25
Difference be- tween over-all heat-transfer rates for dry ice and non- evaporative hemispheres	Kcal/hr.	311	98	43
Difference in total heat transfer to dry ice and non- evaporative hemispheres over a period of 40 sec.	Kca1	3.456	1.088	0.4776
Computed heat transfer by conduction in dry ice hemis-phere in 40 sec.	Kcal	0.05557	0.1066	0.1614
Percentage of total heat transfer dif- ference accounted for by conduction		1.61	9.80	33.80

The conduction effects are more pronounced at the higher Mach numbers at which wind tunnel pressures and temperatures are lowest. The conduction calculation has ignored the effects at the rear of the sphere since it was assumed initially that no ablation occurred in that region. Actually, the base of the sphere is subjected to the lowest temperatures and conduction effects are probably greatest in that region. An upper limit to the total conduction to the entire sphere for a period of 40 seconds has been calculated as:

M	Kcal	Percent of difference in heat transfer between ablating an non-evaporation model		
1.86	0.2003	5.8		
2.84	0.3436	31.6		
4.25	0.4625	96.8		

- 16. In all cases, the dry ice model has been assumed spherical in shape for heat-transfer computations. However, at the lower Mach numbers there is considerable change in the contour of the model during a blow. This change from a spherical front to a more cone-like or to an irregular front would cause an increase in the heat-transfer rates. Also, probably due to the inhomogeneties of the dry ice material, the surface becomes quite rough and again the heat-transfer rates would tend to increase.
- 17. Another possible explanation of the apparently high heat transfer to the ablating sphere is mechanical failure of the model material. At the lower Mach numbers, strong dynamic forces acting upon the dry ice sphere caused loss of mass through surface fragmentation with particles being swept downstream. However, as the Mach number increased, the amount of such visible erosion decreased. Thus, mechanical erosion effects are most important where conduction effects are minimum.

### CONCLUSIONS

18. The determination of heat-transfer rates to subliming dry ice spheres as simply calculated from mass ablation rates yields results which are higher than can be justified by convective considerations. This may be accounted for by: (1) increase in local heat-transfer rates due to surface roughness; (2) increase in heat transfer due to change in model shape; (3) loss of mass without heat transfer through fragmentation; (4) lack of knowledge concerning properties of solid CO2; (5) lack of knowledge concerning vapor pressure around model; and (6) internal heat conduction. Heat conduction effects internal to

the model are significant, especially at the highest Mach number tested, when the greatest change in environmental pressure occurred.

19. Before genuinely conclusive results can be obtained on ablating bodies in supersonic flow, either methods must be devised for obviating the phenomena of mechanical erosion and internal conduction or these effects must be accounted for accurately.

#### REFERENCES

- (a) Gruenewald, Karl H., "Drag and Evaporation Rate of Dry Ice Models in Supersonic Air Flow," NAVORD Report 2954. 21 Sept 1953
- (b) Korobkin, I., "Laminar Heat-Transfer Characteristics of a Hemisphere for Mach Number Range 1.9 to 4.9," NAVORD Report 3841, 10 Oct 1954
- (c) Hodgman, C. D., Handbook of Chemistry and Physics, 34th edition, page 1987, 1952-1953
- (d) Kelly, K. K., Contributions to the data on theoretical metallurgy III. "The Free Energies of Vaporization and Vapor Pressures of Inorganic Substances," Bulletin 383, U. S. Dept. of Interior, Bureau of Mines, 1935
- (e) Plank, R., and Kuprianoff, J., "Die Thermischen Eigenschaften der Kohlensaure im Gasformigen, Flussigen und Festen Zustand," 1929, Beihefte zur Zeitschrift fur die gesamte Kalte-Industrie, Reihe 1, Heft 1
- (f) Smith, J. W., "Effect of Diffusion Fields on the Laminar Boundary Layer," Jour. of Aero. Sci., Vol. 21, No. 3, pp 154-164, March 1954
- (g) Carslaw and Jaeger, "Conduction of Heat in Solids (page 212) University Press, Oxford, Second Impression, 1948
- (h) National Bureau of Standards, Circular Letter LC-763, Oct 1944
- (i) Zeischrift fur die gesamte Dalte-Industrie, pp 109-118, June 1938

#### APPENDIX A

# Heat Conduction Through a Solid Carbon-Dioxide Sphere

1. Considering a solid sphere and assuming that surface conditions do not vary with time.

$$\frac{\partial \mathbf{v}}{\partial \mathbf{t}} = \mathbf{k} \nabla^2 \mathbf{v}$$
 through the solid

 $v = T_1$  initially

 $v = F(\theta)$  at the surface

Let

$$v = u + w \tag{A1}$$

where u is a function of  $\theta$  only and satisfies  $\nabla 2_u = 0$  through the solid;  $u = F(\theta)$  at the surface; and where w is a function of r,  $\theta$  and t such that  $\partial_w/\partial t = K \nabla^2 w$  through the solid;  $w = [g(r,\theta) - u]$  initially; and w = 0 at the surface. Then u is the solution of the steady temperature problem

$$\sum_{n=0}^{\infty} A_n \left(\frac{r}{a}\right)^n P_n(\mu) \tag{2}$$

reference (g). The steady temperature distribution over the surface of the front half of a dry ice sphere satisfied the relation  $F(\theta) = A_0 + A_1 \cos \theta$  where  $A_0$  and  $A_1$  are constant for a given Mach number, see page 6 of text and Figure 3. So that its solution from Equation (2) is

$$u = A_0 P_0(\mu) + A_1 \left(\frac{r}{a}\right) P_1(\mu)$$
 (3)

since  $\mu = \cos \theta$ ,  $P_O(\mu) = 1$  and  $P_I(\mu) = \cos \theta$  and  $\gamma$  has values of 0 and 1 only.

$$g(r,\theta) = T_1 - u = (T_1 - A_0)P_0(\mu) - A_1(\frac{r}{a})P_{\Phi}(\mu)$$
(4)

w is the solution of the problem of variable temperatures with zero surface temperature, reference (g).

$$W = \sum_{n}^{\infty} \sum_{\alpha_{n_{i}}} e^{-K\alpha_{n_{i}}^{2}t} (\alpha_{n_{i}}r)^{-\frac{1}{2}} J_{n+\frac{1}{2}} (\alpha_{n_{i}}r) P_{n}(u) A_{n}$$
 (5)

where  $\alpha_{ni}$  is a root of  $J_{n+1/2}(\alpha_{ni})$ . Since n has values of 0 and 1 there are two sets of  $\alpha$  's and Equation (5) can be expanded as

$$W = \sum_{\alpha_{0i}}^{\infty} e^{-K\alpha_{0i}^{2}t} (\alpha_{0i}r)^{-\frac{1}{2}} J_{2}(\alpha_{0i}r) A_{0\infty} + \sum_{\alpha_{1i}}^{\infty} e^{-K\alpha_{1i}^{2}t} (\alpha_{1i}r)^{-\frac{1}{2}} J_{3}(\alpha_{1i}r) P_{i}(u) A_{i\infty}$$
(6)

where
$$A_{od} = \left\{ a^{2} \alpha_{o}^{-\frac{1}{2}} \left[ \int_{\frac{1}{2}}^{r} (\alpha_{o} r)^{2} \right]^{-\frac{1}{2}} \int_{\frac{1}{2}}^{\infty} \left[ (\alpha_{o} r) dr \right] dr \int_{-r}^{r} g(\theta_{r}) P_{o}(u) du$$
(7)

$$A_{1a} = \left\{ a^{2} \alpha_{1}^{-1/2} \left[ J_{3/2}^{\prime}(a,r)^{2} \right] \right\} \int_{0}^{a} r^{3/2} J_{3/2}(\alpha,r) dr \int_{-1}^{1} g(\theta,r) P_{1}(\mu) d\mu$$
(8)

$$A_{o\alpha} = \frac{2(T_i - A_o) \int_{3/2} (\alpha_o \alpha)}{(\alpha_o \alpha)^{1/2} \left[ \int_{1/2} (\alpha_o \alpha) \right]^2}$$
(9)

$$A_{1\alpha} = \frac{-2A_{1}J_{2}(\alpha_{1}\alpha)}{(\alpha_{1}\alpha)^{1/2}[J_{3/2}'(\alpha_{1}\alpha)]^{2}}$$
 (10)

Adding Equations (3) and (6) give

$$V = U + W = A_0 + \frac{A_1}{a} (C_0 \theta) r + \sum_{\alpha_{0i}} \left[ A_{0\alpha} e^{-K\alpha_{0i}^2 t} (\alpha_{0i} r)^{-\frac{1}{2}} \int_{y_2} (\alpha_{0i} r) \right]$$

$$+ \sum_{\alpha_{1i}} \left[ A_{1\alpha} e^{-K\alpha_{1i}^2 t} (\alpha_{1i} r)^{-\frac{1}{2}} \int_{3/2} (\alpha_{1i} r) C_{00} \theta \right]$$
(11)

The heat transferred at the sphere surface, (r=a), can be expressed by

$$Q = -k \left(\frac{dv}{dr}\right)_{r=a} = -k \left[B_{i} \cos \theta - \sum_{\alpha \neq i} B_{\alpha} e^{-k \cdot \alpha_{i} \cdot t} \sum_{\alpha \neq i} B_{i\alpha} e^{-k \cdot \alpha_{i} \cdot t} C_{r} \theta\right]$$
where  $B_{i} = A_{i}$ ,  $B_{\alpha\alpha} = A_{o,c} \left(\frac{\alpha_{o}}{a}\right)^{1/2} J_{\frac{3}{2}}(\alpha_{o}\alpha)$  and  $B_{i,\alpha} = A_{i,\alpha} \left(\frac{\alpha_{i}}{a}\right)^{1/2} J_{\frac{3}{2}}(\alpha_{i}\alpha)$ 

For a hemisphere,

$$Q = \int_{0}^{A} q dA = \int_{0}^{\frac{\pi}{2}} 2\pi a^{2} q \sin \theta d\theta \quad \text{so that}$$

$$Q = -k \pi a^{-1} \left[ B_{i} - 2 \sum_{\alpha_{i}} B_{\alpha_{i}} e^{-K\alpha_{i}^{2} t} - \sum_{\alpha_{i}} B_{i} \alpha_{i} e^{-K\alpha_{i}^{2} t} \right]$$
(13)

or

$$Q = -2\pi \pi a \left[ A_i - 4(T_i - A_o) \sum_{\alpha_{0i}} e^{-K\alpha_{0i}^{2}t} + 2A_i \sum_{\alpha_{ii}} e^{-K\alpha_{ii}^{2}t} \right]$$
(14)

Heat conducted through a hemisphere in  $t_1$  seconds can be obtained by integrating Q with respect to t through the interval 0 to  $t_1$ 

$$\int_{\delta}^{t_{i}} Q dt = -k\pi a \left\{ A_{i}t_{i} - A(T_{i} - A_{o}) \sum_{\alpha o_{i}} \left( \frac{1 - e^{-K\alpha o_{i}t_{i}}}{K\alpha o_{i}} \right) + 2A_{i} \sum_{\alpha i_{i}} \left( \frac{1 - e^{-K\alpha i_{i}t_{i}}}{K\alpha i_{i}^{2}} \right) \right\}$$
(15)

Considering a whole sphere

$$Q = \int_{0}^{\pi} 2\pi a^{2} g \sin \theta d\theta = 4 k \pi a^{2} \sum_{\alpha o_{i}} B_{o\alpha} e^{-i\alpha o_{i}} t$$

$$Q = 8 k \pi a (T_{i} - A_{o}) \sum_{\alpha o_{i}} e^{-K\alpha o_{i}} t$$

$$Q = 8 k \pi a (T_{i} - A_{o}) \sum_{\alpha o_{i}} e^{-K\alpha o_{i}} t$$

$$Q = 8 k \pi a (T_{i} - A_{o}) \sum_{\alpha o_{i}} e^{-K\alpha o_{i}} t$$

$$Q = 8 k \pi a (T_{i} - A_{o}) \sum_{\alpha o_{i}} e^{-K\alpha o_{i}} t$$

and for a period of 
$$t_1$$
 seconds
$$\int_{0}^{t} Q dt = 8 k T a \left( T_i - A_o \right) \sum_{\alpha_{oi}} \left( \frac{1 - e^{-K\alpha_{oi}^2 t}}{K\alpha_{oi}^2} \right)$$
(17)

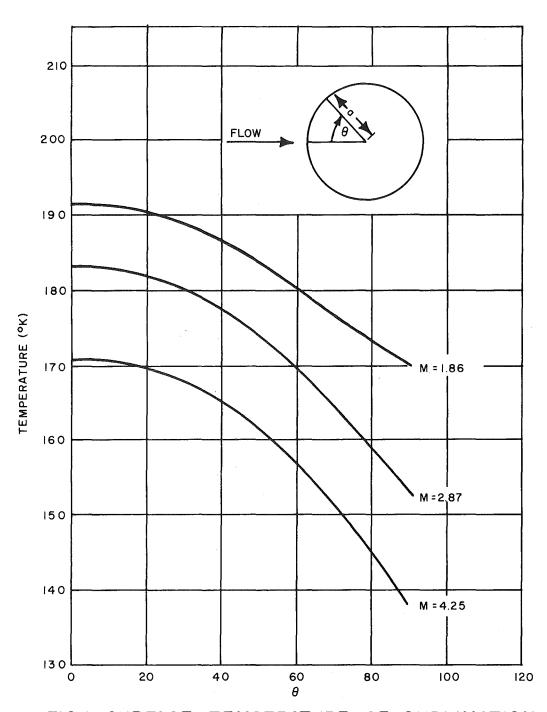


FIG.I SURFACE TEMPERATURE OF SUBLIMATION VS. ANGULAR POSITION ON DRY ICE SPHERE

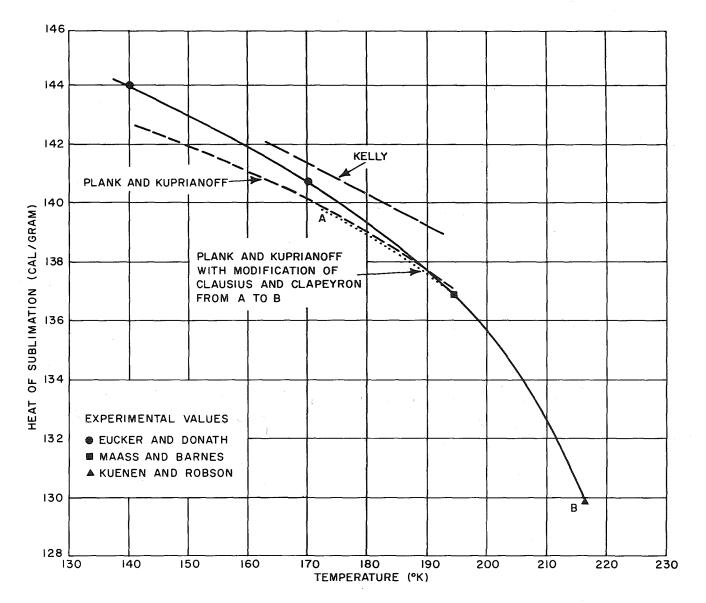


FIG. 2 HEAT OF SUBLIMATION FOR SOLID CO2

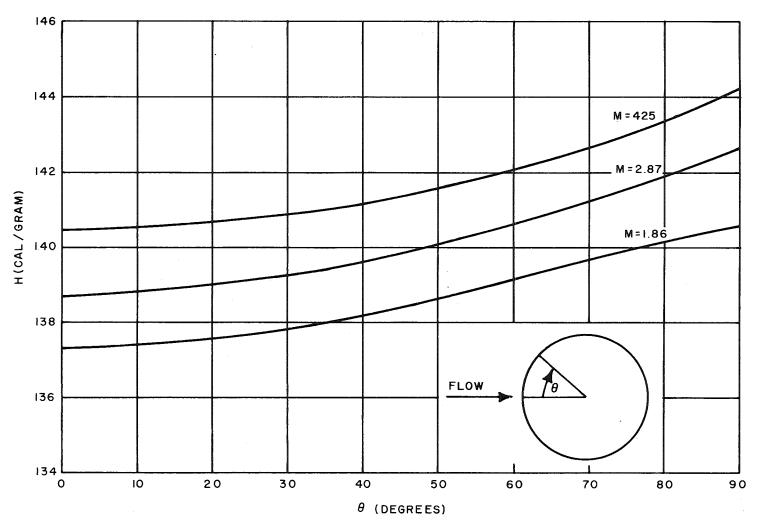
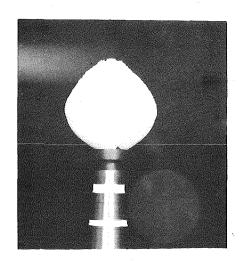


FIG.3 HEAT OF SUBLIMATION OVER A DRY ICE SPHERE IN SUPERSONIC FLOW



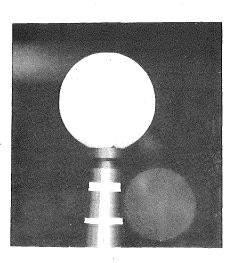


FIG.4 DRY ICE SPHERE, BEFORE AND AFTER BLOWING M=1.86

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